UNIVERSITY OF ESSEX

Undergraduate Examinations 2014

CANDIDATE'S EXAMINATION NUMBER:

MATHEMATICS FOR COMPUTING

Time allowed: **TWO** hours

The following items are provided: formulae (on page 2).

Candidates are permitted to bring into the examination room:
Hand-held, non programmable calculators (containing no textual information)

Candidates must answer **ALL** questions.

The paper consists of **EIGHT** questions.

The questions are not of equal weight.

Answers must be written in the spaces provided on the question paper. If additional space is needed, attach additional sheets firmly to the question paper.

The percentages in brackets provide an indication of the proportion of the total marks for the **PAPER** which will be allocated.

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Total mark for paper:	
Checked by:	

FORMULAE

Provided for the assistance of the candidate.
Not all formulae may be needed in this paper.
The numbers to the left of each formula are for reference and may be used in answers to indicate which formula has been used.

F1 $\neg\neg p \equiv p$

F2 $\neg F \equiv T$

F3 $\neg T \equiv F$

F4 $p \vee q \equiv q \vee p$

F5 $p \wedge q \equiv q \wedge p$

F6 $p \vee \neg p \equiv T$

F7 $p \wedge \neg p \equiv F$

F8 $p \vee p \equiv p$

F9 $p \wedge p \equiv p$

F10 $p \vee T \equiv T$

F11 $p \wedge T \equiv p$

F12 $p \vee F \equiv p$

F13 $p \wedge F \equiv F$

F14 $\neg(p \vee q) \equiv \neg p \wedge \neg q$

F15 $\neg(p \wedge q) \equiv \neg p \vee \neg q$

F16 $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

F17 $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

F18 $p \vee (p \wedge q) \equiv p$

F19 $p \wedge (p \vee q) \equiv p$

F20 $p \rightarrow q \equiv \neg p \vee q$

F21 $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

F22 *k* objects from *N*, <u>order important</u>:

$$\frac{N!}{(N-k)!}$$

F23 $\binom{N}{k} = \frac{N!}{(N-k)!\,k!}$

F24 $\boldsymbol{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad \boldsymbol{A}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

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Candidates must answer ALL questions.

<u>Question 1</u>

De Morgan's Law can be extended to three variables: $\neg(p \wedge q \wedge r) \equiv \neg p \vee \neg q \vee \neg r$

(a) Construct a truth-table to show that the above identity is correct.

[5%]

(b) Show <u>algebraically</u> that the identity is correct. Make each step clear in words, or by quoting the formula number from page 2 (e.g. F3).

[8%]

Question 2

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The **conjunctive normal form (CNF)** is the *dual* of the disjunctive normal form. Given the following proposition in conjunctive normal form:

$$f(a,b,c)=(a\vee b\vee c)\wedge(a\vee\neg b\vee c)\wedge(a\vee\neg b\vee\neg c)\quad ,$$

(a) complete the following truth table for the proposition. Note that the omission of columns for intermediate results is deliberate – you are expected to construct the final column directly from the proposition by adapting what you know about the **disjunctive normal form**. [4%]

a	*b*	*c*	*f(a,b,c)*
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

(b) Show that the CNF above can be simplified to $a\vee(c\wedge\neg b)$ using algebraic steps. Make each step clear in words, or by quoting the formula number from page 2 (e.g. F3). [9%]

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Question 3

A student plans to visit London for the day, and chooses three places to visit represented here by A, B and C. He/she starts from Liverpool Street station and uses a public transport route between each place. There are two routes between Liverpool Street station (LST) and A, four between LST and B, and only one between LST and C. There are three routes between A and B, two between B and C, and four between A and C.

(a) In how many different orders can the student visit the three places? Explain how you arrive at your answer as well as giving a numerical result. [3%]

(b) Explain how to calculate the number of different itineraries which are possible. (An itinerary is an ordering of A, B and C, and a choice of routes between each pair of places which are adjacent in the ordering, including the choices of routes from LST to the first place and from the last place back to LST.) [4%]

(c) Calculate the number of different itineraries using the method described in part (b), or otherwise.

[5%]

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Question 4

In the second assignment, you were required to generate all the ordered combinations of k letters from a given word of N letters. A good way to do this is to generate all the k-letter *unordered* combinations, and then generate permutations from each of the combinations.

(a) Show that the number of combinations generated in this way agrees with formula F22. [4%]

(b) An alternative approach is to create all the permutations of the N-letter word, then extract from the permutations the ordered combinations. There are at least two reasons why this approach is poor compared to the approach described above. What are they? [8%]

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Question 5

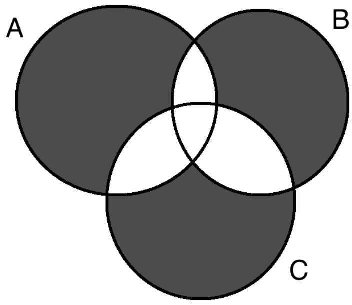

Figure 5.1

(a) Write an expression for the set represented by the shaded area in Figure 5.1. [5%]

(b) Draw a Venn diagram in the box below to represent the set $A \cap (\overline{B \cup C})$ [4%]

(c) Given two sets *C* and *D*, if $C \cup D = C \cap D$ what can you say about *C* and *D*? [4%]

Question 6

For marker's use only

In the laboratory, you used MATLAB to simulate probability. A student is given the task of simulating a biased coin and decides to use the following function code to simulate one toss of the coin, where **tf** is a logical result, and **p** is the probability of the coin landing heads. For example, if the coin is biased with a 45% probability of heads, then **p** = 0.45.

```
function tf = biased_coin(p)
tf = rand < p;
end
```

(a) Why is it **mathematically** not necessary to pass a parameter to the function containing the probability that the coin will land tails? [3%]

(b) Explain how the statement inside the function generates the correct probability that **tf** will be **true.** [5%]

(c) Explain how to test the function to check that it does give the correct probabilities. [4%]

Question 7

For marker's use only

(a) Two vectors are said to be **orthogonal** if their inner product is zero. Show that the two vectors $U = \begin{pmatrix} 7 & 2 & 3 \end{pmatrix}$ and $V = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$:

confirm that the two vectors are orthogonal. [5%]

(b) A matrix is said to be **orthogonal** if its transpose equals its inverse. Given the matrix $P = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$, calculate the product of P with its transpose and hence find the inverse of P **without using formula F24**.

[5%]

(c) Given your result in part (b), modify P to obtain an orthogonal matrix. [3%]

Question 8

For marker's use only

The following are the equations of three lines in the plane, **two of which are parallel**:

$y = 2x + 1$, $y = -2x + 2$, $y = 2x - 1$

(a) Choose two of these lines **that intersect** and write down a matrix-vector equation in the form **Ax** = **b**.

[3%]

(b) Find the inverse of the matrix **A** that you have written down.

[3%]

(c) **From your results** in parts (a) and (b), find the point of intersection of the two lines. [3%]

(d) **Without reworking the matrix inverse**, find the other point of intersection with the line that you did not choose in part (a). [3%]

END OF PAPER CE141-4-AU